Mathematical Modeling of Quarter Car Suspension Systems on Square road profile

Adam Bin Samsudin[1], Shamsuddin Bin Ahmad[2], Ezzatul Farhain Binti Azmi[1]

[1)] Faculty of Engineering Technology, Universiti Teknikal Malaysia Melaka,
75450 Ayer Keroh, Melaka, Malaysia
[2)] Faculty of Science, Universiti Teknologi Malaysia,
81310 Skudai, Johor, Malaysia

[*]Corresponding e-mail: adam.samsudin@utem.edu.my

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ABSTRACT – Suspension systems have been widely applied to vehicles. It gives a comfortable ride to the driver through many type of road profile. A simple mathematical model of a passive quarter car suspension system has been formulated analytical to determine whether the suspension of car model will give a comfortable ride to a square road profile. This study is using Ford Scorpio car model data that are being applied with considering the basic quarter car model and the refinement of basic quarter car model.

1. INTRODUCTION

To maintain an excellent ride and smooth control while driving, a suspension system has been applied to vehicle. A good suspension system should provide a comfortable ride and good handling within a reasonable range of deflection [1]. Basically, the main function of car suspension system is to keep the car's wheels in firm contact with the road [2]. This may look simple, but with an acceleration comes force, and forces it to translates into raw energy, when a vehicle accelerates down a road bumps cause forward energy to be converted into vertical energy, which travels through the frame of the vehicle. Without coil and springs to absorb this, vertical energy would cause vehicles to jump up off the road, reduce tire friction and control. The car will then come bounding back down with even greater force, creating for a very uncomfortable and dangerous ride. As a significance, the car suspension system essentials to be studied and suspension for car also has their possible to improve vehicle performance.

2. METHODOLOGY

In this study, we formulate an elementary quarter car suspension model on a 'square road profile'. Numerical solution of 4th order Runge-Kutta methods are used in Maple 13 to solve the suspension system. Where, the result of car suspension system for simple and refinement model can be shown graphically.

1.1 Model analysis

Figure 1 and Figure 2 shows the force diagram of simple and refinement car model where we are able to formulate a mathematical suspension system with satisfying Newton's second law of motion.

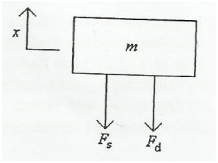

Figure 1: Force diagram of simple car

By applying the Newton's second law of motion, the equations of the simple car motion can be found as in Equation (1).

$$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y} \quad (1)$$

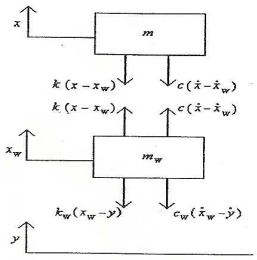

Figure 2: Force diagram of refinement car

And, the equations of the refinement simple car motion can be converted into Equation (2) and Equation (3).

$$m\ddot{x} = -k(x - x_w) - c(\dot{x} - \dot{x}_w) \quad (2)$$

$$m_w\ddot{x}_w = k(x - x_w) + c(\dot{x} - \dot{x}_w) - k_w(x_w - y) - c_w(\dot{x}_w - \dot{y}) \quad (3)$$

1.2 Square road profile

Assume that the car is travelling with an average horizontal speed. Thus, the equation of the 'square' road profile discovered as:

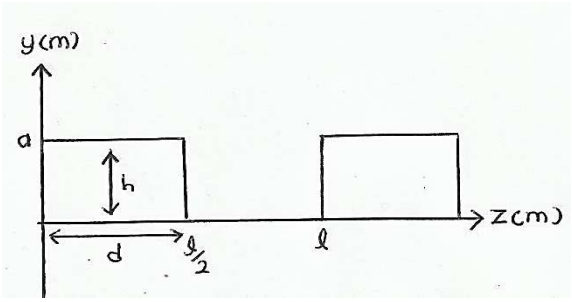

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Figure 3: Square road profile

$$y = f(t) = \begin{cases} a, & 0 < x < \frac{l}{2} \\ 0, & \frac{l}{2} < x < l \end{cases}$$

3. RESULTS AND DISCUSSION

The good suspension system should satisfy the prediction by graph,

I. $|x| \leq 0.1$ for all value of time, t.

II. $|\ddot{x}| \leq 0.6g$ for all value of time, t.

3.1 Suspension of simple car model

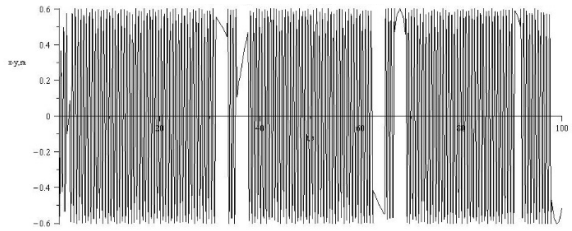

Figure 3: Graph of displacement x againt t.

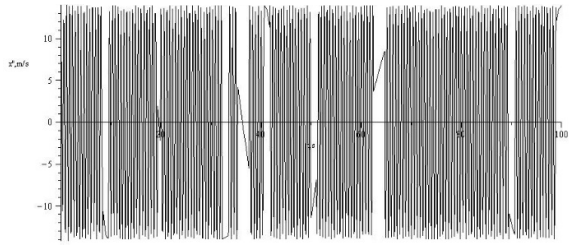

Figure 4: Graph of acceleration x'' againt t.

3.2 Suspension of refinement car model

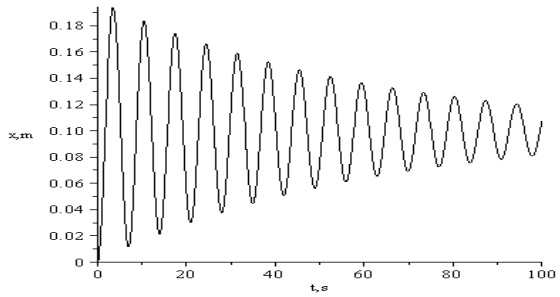

Figure 5: Graph of displacement x againt t.

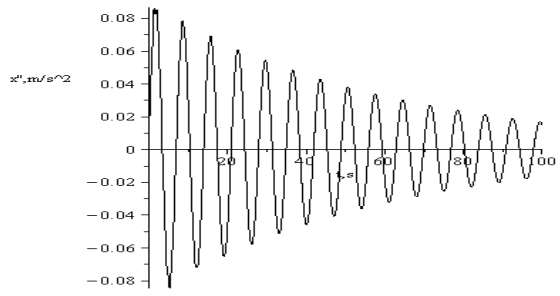

Figure 6: Graph of acceleration x'' againt t.

The graph of vertical displacement of simple and refinement car model are indicating in figure 3 and figure 5. While, Figure 4 and Figure 6 indicate vertical acceleration of simple and refinement car model. Both plotting graph of displacement and refinement for simple car model from figure 3 and 4 shows that it bound not implied the prediction in I.

Whilst, the graph of acceleration for simple car also give a huge difference from the prediction in II. The suspension of refinement car give a smaller value compared to simple car model and both graph for refinement model show decrease changes lead to show that refinement model gives a better suspension. Whereas, the displacement and acceleration involves from simple model shows it oscillate unchanged in 100 second.

From both plotting graph, the number of oscillation for refinement car model is less than the number oscillation of simple car model. It directed that, the number of damping is fewer for refinement car model and indicate better suspension. However, the graph of displacement for refinement car model only oscillate on positive side show that the spring are not able to stabilizer car body when move on square road profile.

4. CONCLUSION

In conclusion, the study of car suspension system on square road profile found that the refinement car model by adding axle and wheel to the simple car model indications that it is also not fulfilled the prediction to be more comfortable ride compared to simple car model. Nevertheless, it is shows that refinement car model still give a comfort ride compared to simple car model.

5. REFERENCES

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